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IS *MARS'S* AXIAL VELOCITY CHANGING?

By A. M. Mattoon.

The main principles of the Nebular Hypothesis were published about a century ago. LAPLACE reasoned skillfully regarding the behavior of a contracting mass, such as that of which he believed the solar system to have been composed. He described the processes by which successive concentric rings would become detached from the central body of gas, and the material ultimately be gathered together to form the planets. The equatorial telescope has done little or nothing to confound the argument of the great astronomer. *Saturn's* rings may be deemed incompetent witnesses to sustain his theory, on the grounds that they consist of countless myriads of minute moons, already solidified, and are no longer in the nebulous state. But surely *Andromeda* affords an object-lesson not liable to this criticism. Here on gigantic scale we have *Saturn* reproduced — this time in the very material considered by the noted Frenchman.

But some will, no doubt, maintain that processes different from the ring formation may have brought a planet out of chaos. JACOBI and POINCARÉ and the younger DARWIN have told us of the probability of a different order of events. They have argued that a rotating nebula may readily become a prolate spheroid, the axis of whose motion is its shortest diameter. They have reasoned that with increase of speed this ellipsoid must *necessarily* change its form into that of a pear-shaped body, called the apioid, and this in turn modify its outline till it becomes a huge dumb-bell, in which the globes are widely discrepant in size. According to a law of physics, a line drawn on the surface from one pole of a rotating body to the other, will describe equal areas in equal times, provided the body is unobstructed in its axial motion. Then, if condensation of the nebula result in contraction, *as it must*, the increase of speed would ultimately produce so heavy a strain upon the slender neck uniting the two balls as to sunder it; and thus a world would set up its independence from the Sun, though still yielding him a loyal allegiance in becoming his faithful attendant.

We shall have no occasion to quarrel with any of these eminent scientific men regarding the history of the bodies composing the solar system, and the manner of their building. Both may be right, for aught we need to know in dealing with our present question. It is certain that planets abound; and the remarkable harmony which they display regarding the direction of their axial motion at least suggests that once there was some sort of physical connection. Prophecy is not the province of this paper; hence no prediction is ventured concerning what the future will disclose with reference to the rotation of *Uranus* and *Neptune*. At least six planets, as well as the central luminary of the solar system, turn on their axes in the same direction.

But what about the uniformity of the *rate of rotation* these bodies have continued to maintain? Have any of them changed their axial velocities? According to a law already quoted, if they have contracted, that would cause their rotation to become *increasingly* rapid. But is there some mysterious brake, somewhere in space, that can operate to *diminish* the speed of motion on the axis? It is hardly possible that *Mercury* and *Venus* should not once in the ages of antiquity have been turning faster than at present. How is it that now they are keeping the same face toward the Sun? YOUNG, in his "General Astronomy," has said regarding the rotation of *Jupiter:* "The great red spot has given values ranging from $9^h\ 55^m\ 34^s.9$ (in 1879) to $9^h\ 55^m\ 40^s.7$ (in 1886), the time of rotation as determined in each case being certainly accurate within half a second." If the Princeton astronomer is right, here is a lengthening of the day by nearly six seconds in seven years.* This is a question of *fact*, and not of *theory;* and its answer is within the reach of the telescopes of the present day. Has the Sphinx propounded another riddle? The friction of the tides may be suggested as a possible explanation. But what about the neutralizing influence upon the tides, when satellites are scattered about profusely? The query naturally arises, whether it is possible that *Jupiter's* axial motion will ever diminish, so that the nearest satellite will revolve in a period shorter than that of the planet's rotation. If so, we shall then have the behavior of *Mars* and *Phobos* duplicated. In such a case Mr. DARWIN'S theory of tidal evolution may need some modifications to adapt it to the case in hand. No attempt will here be made to disparage the argument of this eminent English mathematician. No effort is to be put forth to cause any one to doubt that the tides on *Mars* may have influenced the orbital motion of *Phobos*. This discussion will pertain to a question entirely independent of that. The attempt will be made to show that *the day on Mars is gradually growing longer.*

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* It should be noted that different markings on *Jupiter*, even in the same latitude, give different values for the rotation period; and it is at least possible that all of the visible markings are in motion relatively to the planet's surface.

At favorable opposition, the planet is only a little over thirty-five million miles away from us. The opportunities for advantageous observation are frequent enough for astronomers to have some idea of the surface and climates of the various portions of the globe. Enough is known of the polar caps to make it almost absurd to claim that they can be any thing else than snow or ice. They have a diameter almost identical with that of the frigid zones, and have the poles of the planet as their approximate centers. When the northern hemisphere is in a position analogous to that which upon the Earth produces summer, the arctic cap disappears, and the southern one gradually grows to its full proportions. But when half the Martian year rolls round, the northern cap becomes a prominent object, while the one about the opposite pole slowly vanishes. The planet's equator is inclined 24° 50′ to the plane of its orbit; hence its frigid zones are similar to those upon the Earth, except that they are a little larger in proportion to the size of the sphere upon which they are situated. The year upon that globe is a little over twenty-two months long, as we count time; hence the arctic winter must last nearly or quite eleven months. The planet is farther from the Sun than we are; and this would naturally cause the winters there to be colder and longer in proportion to the length of the year than we have them here upon the Earth.

SCHIAPARELLI'S map represents *Mars* as having but one ocean of any real magnitude, and that is situated almost wholly in the southern hemisphere. During the time that the northern snow-cap is forming the Sun's most potent influence is felt just south of the equator, in that part of the ocean lying within the torrid zone. Hence the vapor that is ultimately converted into the snow that goes to form the cap, must come mainly, if not entirely, from the southern hemisphere. Now, this great volume of water is transferred to the polar regions by means of the atmosphere. Here upon the Earth the clouds that become vehicles to transport moisture between places widely separated, do not appear to affect, in the least, the Earth's axial motion. Whatever results might be produced that way, if they are worth considering at all, must be more or less like the friction of the tides, and could hardly operate to accelerate rotation. Indeed, if tides destroy momentum, the broad currents in the great oceans of air would doubtless exert a kindred influence, rather than the opposite. However, nothing is based upon this, other than that the transfer of water from the ocean in the southern hemisphere to the northern polar regions is accomplished without appreciable alteration in the planet's period of rotation. Now, when the moisture, which came from the ocean in close proximity to the equator, left its home upon the sphere, the swiftness of the planet's motion in those regions had given it considerable momentum. Whether or not any of its energy was dissipated into space during the long journey northward, which is not at all improbable, when it settled down again upon the planet, it took up a position so much nearer the axis of the rotating sphere, that the leverage which it had, to act upon that body, was so reduced that it could not possibly restore all that it had taken away. Here in its new position its motion due to the rotation of the globe is very slow. As has been said, the Martian winter in the polar regions would doubtless be not far from eleven months long, nearly twice that of the corresponding seasons in the frigid zones upon the Earth. Accepting Mr. CAMPBELL'S estimate that the atmosphere on *Mars* is not more than one fourth as dense as that to which we are accustomed, and at the same time taking into account the fact that the area covered by the water on the planet just named is only about two thirds as great in proportion to the size of the sphere as the oceanic surface of the Earth, it is certainly far within bounds *to estimate the average depth of snow* in the northern cap, when it attains its maximum, *to be two feet.*

Snow left undisturbed for so long a time as this remains in place, becomes packed extremely hard. Even when the superficial gravity is but a fraction of what it is upon the Earth, such snow would yield a large percentage of its bulk in water, when the advent of spring would cause it to melt. Experiments performed upon snow only moderately compressed show that it will yield about 40 per cent. of its bulk in water. Professor PICKERING, in the October, 1892, number of *Astronomy and Astrophysics*, states that the cap covered the polar regions down to 65° north latitude. That makes its area a zone with a radius of 25°, having the pole as its approximate center. Though the planet is really a *spheroid*, its eccentricity is so small that, for all practical purposes, we shall admit no appreciable error in regarding the cap as covering a zone of a sphere. From geometry, such an area equals the altitude of the zone multiplied by the circumference of a great circle. Supplying the figures for a planet whose radius is 2,100 miles, we obtain as the area of the cap almost exactly 2,600,000 square miles. This area, reduced to square feet and multiplied by the estimated two feet, as the average depth of snow, gives the volume as 145 trillion* cubic feet, which would yield 58 trillion cubic feet of water. We are accustomed in our laboratory experiments to think of water as weighing 62½ pounds to the cubic foot. But upon *Mars* surface gravity is only 38 per cent. as great as here; therefore a cubic foot of water there, weighed by our spring balances, would weigh only 23¾ pounds. *This makes the weight of the cap about 1⅜ quadrillion pounds.*

The planet is an oblate ellipsoid, whose equatorial diameter is about 4,200 miles, and the polar diameter some twenty miles less. The integral calculus gives the volume of such a solid as something over 38½ billion cubic miles, and the exact figures being reduced, make the result a little more than 5⅔ sextillion cubic feet. As has already been stated, one cubic foot of water at the Earth's surface weighs 62½ pounds. The average cubic foot of the Earth weighs 5.58 times as much, or 348¾ pounds. But the density of *Mars* being only 73 per cent., and surface gravity 0.38, one cubic foot of that planet, weighed upon its own surface, would give a result of 96¾ pounds. Multiplying this by the volume in cubic feet, we find that *Mars weighs just a little short of 550 sextillion pounds* — that is, fifty-five with twenty-two ciphers annexed.

Now, if we are to determine the influence of the snow-cap upon the planet, we must separate the weight of each from the other. But the figures last given consist of the total weight of everything on the sphere and within it. Preserving the significant figures in the exact computation down to billions, and deducting the weight of the cap from the total weight, we obtain for *the weight of the planet, exclusive of the cap, about 549⅘ sextillion pounds.*

According to mechanics, momentum is the product of mass and velocity. But the velocity of *Mars* due to rotation is neither that of the swiftest particle, which is situated on the equator, nor that of one located on the axis, where the speed reduces to zero. But there must be a mean, or representative, particle, situated somewhere between these two extremes. Now, either the northern or southern hemisphere may be regarded as generated by the revolution of one fourth of an ellipse about the semi-minor axis. The mean particle in this quarter-ellipse will, of course, reside at its center of gravity. The opposite hemisphere from that treated first, would give as a result for its center of gravity a distance from the minor axis the same as the other, and the representative particle for the *whole planet* would be at the middle of the line joining the two centers of gravity just described. Thus the radius of gyration for *Mars* would have a length equal to the distance of that point from the polar axis of the planet. Summoning the integral calculus again to our assistance, we find the radius sought to be 891.2 miles. This means that during one rotation of *Mars* the representative particle describes a circle whose radius is the length just given. This it does in one Martian day, which, at present is 88,642.67 seconds long. A simple arithmetical calculation will show that its speed is 333.542 feet per second. Since momentum is the product of mass and velocity, and these two factors now are known, *the planet, exclusive of the cap, has a momentum of 183⅜ septillion foot-pounds*, a number consisting of twenty-seven figures.

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* The American and French system of notation will be employed in the figures it becomes necessary to use.

Now, the cap at its maximum covers a zone of one base whose radius is twenty-five degrees, the pole being the center of the same. Let two opposite meridians of the planet be conceived as drawn: then if a plane bisect the sphere in such a way as to include these two meridians, and that plane be extended so as to cut this northern snow-cap into halves, the section formed would be a crescent. The produced axis of the planet would bisect this section. One of these two half-crescents may be regarded as a triangle, two of whose sides are arcs of circles, with sides twenty-five degrees long and described by radii a little greater than the semi-major axis of the planet. The calculus shows that this radius is almost exactly 2,110 miles long. The third side of this triangle is a straight line whose length is equal to the depth of the snow at the pole. While this semi-section of the ice-cap is not a triangle in the common acceptation of that term, for our present purpose it is near enough to it for the law about to be introduced to be applicable. According to mechanics, the representative particle in this quasi-triangle will reside on the medial line (here an arc) one third of the distance from the base to the vertex. This locates it 8° 20′ from the pole. The perpendicular distance of this point from the axis of the planet, according to trigonometry, is 305¾ miles. The velocity of the snow-cap due to the planet's axial motion is that of this mean particle, which in one Martian day, at the present rate, describes a circle whose radius is 305¾ miles, in 88,642.67 seconds. This makes the velocity of the cap 114.4 feet per second. The factors are now both known for finding *the momentum of the ice-cap;* and their product makes it *a little more than 157½ quadrillion foot-pounds, while the material of the cap lies in the arctic regions.*

Reference has been made to SCHIAPARELLI'S map of *Mars.* An examination of it will reveal the fact that a very large proportion of the so-called canals lead from the north polar regions directly to the ocean. Mr. LOWELL, who has given this planet special study, expresses the belief that these markings are stretches of country containing luxuriant vegetation, and that the soil is rendered productive by a central watercourse. He concludes that these channels convey the fluid product of these annually recurring snow-caps back to the ocean. Whether that explanation be the true one or not, if the vapor that becomes snow and goes to form these caps comes *from* the ocean, it must find its way back *to* it again, else it would not take many years to drain it of all its contents. As that body of water is almost wholly in the southern hemisphere, the annual flow of water from the arctic regions must cross the equator to reach the ocean. Then at the crossing-point, this melted snow attains a velocity due to axial motion equal to the swiftest particle on the planet. But it will hardly be fair to compute its momentum for *that* locality, and say that it has gained the difference between *that* and what it had while lying in the frigid zone. When the cap was in process of formation the southern hemisphere had summer; and the evaporation of water that entered into the material composing the cap took place *south* of the equator, mainly in the torrid zone. It will certainly be fair to estimate the vapor as coming from an average distance of 20° south of the equator. To preserve the equilibrium it must necessarily return to the places whence it came. Then, whatever influence this water-flow has for the twenty degrees after crossing the equator would be counterbalanced by that of the last twenty degrees before that line was passed. Hence the ultimate effect would be produced at 20° north latitude. Here the radius of gyration would be 1972.2 miles. The velocity of a point upon the surface at that locality, found just as before, is — at present — 738.1 feet per second. Then, the momentum of the cap, when its material is returned to its original resting-place in the ocean bed, is 1,016¼ quadrillion foot-pounds. But the momentum obtained for it before it left the frigid zone was a little over 157½ quadrillion foot-pounds. Hence the increase in momentum was more than 858 quadrillion foot-pounds. But this gain had to be imparted to it by the planet, or what the cap gained the rest of the planet lost.

It is evident that the material composing both planet and snow-cap together is not a variable quantity — that is, the joint mass is constant. Now, since momentum is the product of mass and velocity, if the momentum suffer a decrease, the result must be a diminished velocity. To find what this factor is under the new conditions, we must divide the joint momentum by the total mass. Supplying the exact figures, and performing the indicated division, the quotient obtained is 333.54177166 feet per second. Now, this is the velocity of the mean particle which is, as before shown, 891.2 miles from the axis of rotation. When particles describe the same circle at different velocities, their times are inversely proportional to their velocities. Hence the new velocity is to the old as the old period is to the new. Substituting values, we have the proportion —

$$333.54177166 : 333.542 :: 88{,}642.67 : 88{,}642.76.$$

From this we see that the day has been lengthened by 0.09 of a second by one transfer of the material of the snow-cap from the ocean to the north pole and back again. The southern cap doubtless influences axial velocity to some extent; but the conditions under which it acts are very different from those of the northern one, and cannot be discussed here. If the northern cap produces the effect here indicated, it would require only 680,000 Martian years, or 1,275,000 of our years, to change the length of the day by 17 hours. This is equivalent to saying that so long ago the day there might have been only $7^h\ 37^m\ 22^s.67$ long, which is shorter than the orbital period of *Phobos*, the inner satellite of *Mars*.

The reader should not receive the impression from the preceding argument that the writer considers it established that

680,000 years ago, or even twice or thrice that period, *Mars* was rotating at any such rate of speed as indicated in the foregoing figures. That there have been, and still are, influences at work to counteract, *in part*, the effects of these spring freshets, has not escaped notice.

In the first place, as the water returns from the frigid zone and flows into the northern portion of the ocean bed, the water-level would be preserved by a sort of tide, setting in toward the south pole. And the eastward motion of this tide, due to the planet's rotation, being greater than the velocity of the antarctic regions, toward which it approaches, would tend to accelerate rotation. But it will be apparent that the next time evaporation takes place to form a second northern snow-cap the water comes from the same regions as before; and a northward flowing tide, to fill up the place of that which was evaporated, would counteract the influence of the former tide-wave.

In the second place, there is doubtless some influence exerted by the vapor, as it is suspended in the atmosphere while it is making its journey northward from the ocean bed to its place of long repose in the polar regions. The eastward impetus that rotation had given it would make its motion swifter than that of the lands beneath it as it approached the frigid zone, and a northeast wind would be the result. Though trade-winds do not seem to have any influence upon the Earth's rotation, it may be possible that the friction of this vapor-laden atmosphere would slightly accelerate the axial motion of *Mars*.

In the third place, all the writers on cosmogony agree that the planets began independent existence in a very heated condition, and gradually cooled, by the radiation of their heat. When *Mars* first experienced a snowfall at the pole, the quantity must indeed have been quite insignificant. This depth of two feet which has been employed in our estimate was not attained, till snow had been falling there perhaps a million years or more. This fact does not, of course, argue against what has been claimed. It merely goes to show that the forces that have been at work to destroy the planet's axial motion, have needed longer time to accomplish their results.

And now perchance some one may say that the material of the snow-cap does not come from the ocean, but from some other source. As to that, it must come from *somewhere*. Evaporation of water must take place where it is warm enough to cause it.

The snow cannot have come from the pole, nor from any source beneath the cap. Snow descends from the atmosphere. It cannot have come from the cap itself; for that does not exist for about half the year. It must have had its origin from some body of water nearer the equator than itself. In any such case, the argument of the foregoing computation maintains its force; for the water of which it is composed must return to the place whence it came, else the supply would soon be exhausted.

So it would appear, at least to an amateur in the study of astronomy, such as the writer frankly confesses himself to be, that, when the material of the Martian snow-cap moves *from a position near the axis of rotation to one more remote from it, momentum must be lost.* As the counteracting influences all combined are insufficient to restore what has been taken away, axial velocity must be diminished, and the day on *Mars* must be slowly but surely growing longer.

PARK COLLEGE, PARKVILLE, MO., December, 1898.

THE CONSTANT OF ATTRACTION.

BY FREDERICK H. SEARES.

In view of the confused designations of the constant of attraction as given by many writers upon theoretical astronomy, it is perhaps worth while for the benefit of students to call attention to its true significance.

NEWTON'S law of attraction states a proportionality between the force of attraction exercised by two bodies, on the one hand, and the product of their masses divided by the square of the distance between them, on the other. In practice it is convenient to express this law in mathematical symbols in the form of an equation, the proportionality being converted into an equality by the introduction of a constant factor whose numerical value depends upon the numerical values employed for the fundamental units. Assuming the law of gravitation to be accurately true for all parts of space, this factor, which may be called the constant of attraction, is a true constant of the universe. Its value changes only with a change in the fundamental units.

The law of attraction therefore takes the form

$$\text{Force} = A\,(\text{mass}) \times (\text{acceleration}) = \frac{C\,(\text{mass}) \times (\text{mass})}{(\text{distance})^2} \qquad (1)$$